



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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2 (a) Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^n r(r+1)(r+2)$ in terms of n , fully factorising your answer. [3]

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- (b) Express $\frac{1}{r(r+1)(r+2)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^n \frac{1}{r(r+1)(r+2)}.$$
 [5]

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- (c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$. [1]

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3 The sequence of real numbers a_1, a_2, a_3, \dots is such that $a_1 = 1$ and

$$a_{n+1} = \left(a_n + \frac{1}{a_n}\right)^3.$$

(a) Prove by mathematical induction that $\ln a_n \geq 3^{n-1} \ln 2$ for all integers $n \geq 2$. [6]

[You may use the fact that $\ln\left(x + \frac{1}{x}\right) > \ln x$ for $x > 0$.]

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(b) Show that $\ln a_{n+1} - \ln a_n > 3^{n-1} \ln 4$ for $n \geq 2$. [2]

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4 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix \mathbf{M} represents a sequence of two geometrical transformations.

State the type of each transformation, and make clear the order in which they are applied. [2]

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(b) Find the values of θ , for $0 \leq \theta \leq \pi$, for which the transformation represented by \mathbf{M} has exactly one invariant line through the origin, giving your answers in terms of π . [9]

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5 The plane Π has equation $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j})$.

- (a) Find a Cartesian equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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The line l passes through the point P with position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the vector \mathbf{k} .

- (b) Find the position vector of the point where l meets Π . [3]

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(c) Find the acute angle between l and Π .

[3]

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(d) Find the perpendicular distance from P to Π .

[3]

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(b) Sketch C .

[2]

(c) Find the area of the region bounded by C and the initial line, giving your answer in exact form. [6]

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7 The curve C has equation $y = \frac{4x+5}{4-4x^2}$.

(a) Find the equations of the asymptotes of C . [2]

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(b) Find the coordinates of any stationary points on C . [4]

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(c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

(d) Sketch the curve with equation $y = \left| \frac{4x+5}{4-4x^2} \right|$ and find in exact form the set of values of x for which $4|4x+5| > 5|4-4x^2|$.

[6]

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